Problem 1 (60 pts). Consider the following schema:

STUDENT(Id, StName, Age)
TRANSCRIPT(StudId, CrsCode, Semester, Grade)
TEACHING(ProfId, CrsCode, Semester)
PROFESSOR(Id, ProfName, Dept, Sal)

1. (15 pts) Raise by 10% the salary of every professor who is now younger than 40 and who taught CSE132B in the Spring 2002 or Fall 2003 semester.

2. (15 pts) Find the professors whose salaries are at least 10% higher than the average salary of all professors in their departments.
3. (15 pts) Write the following query in SQL:

Find the Ids of all students who have taken a course from each professor in the CS Department.

4. (15 pts) Define the above query as an SQL view and then use this view to answer the following query:

For each student who has taken a course from every professor in the CS Department, show the number of courses taken, provided that this number is more than 10.
Problem 2 (30 pts). Suppose we have a database for an investment firm, consisting of the following attributes: \( B \) (broker), \( O \) (office of a broker), \( I \) (investor), \( S \) (stock), \( Q \) (quantity of stock owned by an investor), and \( D \) (dividend paid by a stock), with the following functional dependencies:

\[
S \rightarrow D, I \rightarrow B, IS \rightarrow Q, B \rightarrow O.
\]

1. (10 pts) Find a minimal key for the relation scheme \( R = BOSQID \) and prove it is a key.

2. (10 pts) How many minimal keys does relation scheme \( R \) have? Prove your answer.

3. (10 pts) Suppose we decomposed relation \( R \) into the two relations \( R_1 = ISQD \) and \( R_2 = IBO \). Is this a BCNF decomposition? Is it a 3NF decomposition? Justify your answers.
Problem 3 (10 pts). Are the following rules about FDs true?

- If $A \rightarrow B$ then $B \rightarrow A$.

- If $AB \rightarrow C$ and $A \rightarrow C$, then $B \rightarrow C$.

- If $AB \rightarrow C$, then $A \rightarrow C$ or $B \rightarrow C$.

Prove your claim (no partial credit if you correctly guess “yes” or “no” without substantiation). If you answer “yes”, prove it by checking that the FD to the right of the “then” is implied by the FDs to its left. If you answer “no”, then give a counter-example relation that satisfies the FDs to the left of the “then”, but violates the ones to its right.